THE INTERRUPTED POWER LAW AND THE SIZE OF SHADOW BANKING

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Abstract. Using public data (Forbes Global 2000) we show that the distribution of asset sizes for the largest global firms follows a Pareto distribution in an intermediate range that is "interrupted" by a sharp cutoff in its upper tail, which is totally dominated by financial firms. This contrasts with a large body of empirical literature which finds a Pareto distribution for firm sizes both across countries and over time. Pareto distributions are generally traced back to a mechanism of proportional random growth, based on a regime of constant returns to scale: this makes our evidence of an "interrupted" Pareto distribution all the more puzzling, because we provide evidence that financial firms in our sample operate in such a regime.

We claim that the missing mass from the upper tail of the asset size distribution is a consequence of shadow banking activity and that it provides an estimate of the size of the shadow banking system. This estimate – that we propose as a shadow banking index – compares well with estimates of the Financial Stability Board until 2009, but it shows a sharper rise in shadow banking activity after 2010.

1. Introduction

If we take Forbes Global 2000 list[1] as a snapshot of our global economy, we find that financial firms dominate the top tail of the firm distribution by asset size: the first non-financial firm is General Electric, which ranks 44st in the 2013 Forbes Global 2000 (FG2000) list. General Electric is also the first non-financial firm in the 2013 Fortune 500 list[2], which covers only the US economy, where it ranks 11th. This seems to be a recent trend: General Electric was the largest non-financial firm by asset size also in the 2004 FG2000 list and in the Fortune 500 list of 1995, but

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[1] See http://www.forbes.com/global2000/list/. Notice that the list refers to the previous year. Thus the 2013 FG2000 list collects firms according to their characteristics in 2012. In the present paper the financial sector includes all the firms that in the FG2000 list belong to the following industries: Banking, Diversified Financials, Insurance, Consumer Financial Services, Diversified Insurance, Insurance Brokers, Investment Services, Major Banks, Regional Banks, Rental & Leasing, Life & Health Insurance, Thrifts & Mortgage Finance, Property & Casualty Insurance. Their number ranges from 501 in 2013 list to 597 in 2008 list.

[2] See http://money.cnn.com/magazines/fortune/fortune500/2013/full_list/.

then it ranked 22nd and 3rd respectively[3]. Financial firms are approximately 30% of the firms in the FG2000 list and they account for approximately 30% of the total sales, profits and market value, a share that has been roughly constant in the whole period 2003-2012 studied. Yet, financial firms account for 70% of total assets in the 2004 FG2000 list, a share that rose to 87% in the 2013 list[4].

The size of the biggest financial firms, besides being remarkable, also displays a peculiar distribution: the 12th largest firm in the 2013 FG2000 list, Royal Bank of Scotland, has 2.13 trillion \$ in assets (which is comparable to the gross domestic product of its country of origin; the UK has indeed a GDP of 2.4 trillion \$), but its size is not much smaller than the largest firm in the list, Fannie Mae, that has assets worth 3.2 trillion \$.

This observation contrasts with the common view in the literature documented across countries and over time (see Axtell (2001), Fujiwara (2004), and Gabaix (2009)) that firm sizes S follow a Paretian distribution as

$$\text{Prob}\{S \geq x\} \simeq cx^{-\gamma}, \tag{1}$$

with $\gamma, c > 0$.

Fig. 1 shows that the rank plot of the firms included in the 2004, 2007 and 2013 lists of FG2000 follows Eq. (1), with an exponent γ close to one, corresponding to Zipf's law (see Axtell (2001)) only from the 20th largest company downward[5]. The upper tail, which is entirely dominated by financial firms, levels off. If Zipf's law were to hold also for the top 20 companies, we would expect Fannie Mae to be ten times as large as the Royal Bank of Scotland (21.3 instead of 3.2 trillion \$).

This anomaly in the shape of the top tail of the assets distribution is the starting point of our analysis.

From a theoretical point of view, the occurrence of power laws (i.e. Pareto distributions) in the size distribution of firms has been related to proportional random growth (PRG) models (see Gabaix (2009)). Section 2 enquires whether departures from the PRG model's prediction may be due to an anomalous dynamics of financial firms that dominate the upper tail of the distribution. Our conclusion is that the available data suggest that PRG should hold for financial firms. The analysis therefore provides a theoretical framework which allows us to calculate the hypothetical assets distribution in the absence of any anomaly. Section 3 argues that the difference between this hypothetical distribution and the actual one can be taken as a proxy of the size of the so-called *shadow banking system*, which

[3]2004 is the first year we could find for the FG200 list and 1995 is the first year when Forbes 500 began to include also financial firms.

[4]Non financial firms in FG2000 totaled approximately 20 trillion \$ both in 2004 and 2013 lists, whereas the total assets of financial firms in FG2000 list steadily soared from 48 trillion \$ in 2004 to 138 trillion \$ in 2013, almost doubling world's GDP. This trend is called *financial deepening* in Haldane (2012) to which we refer for a discussion on the systemic implication of the growth in the size of banks.

[5]All computations are made in R (2013). All datasets and codes are available upon request.

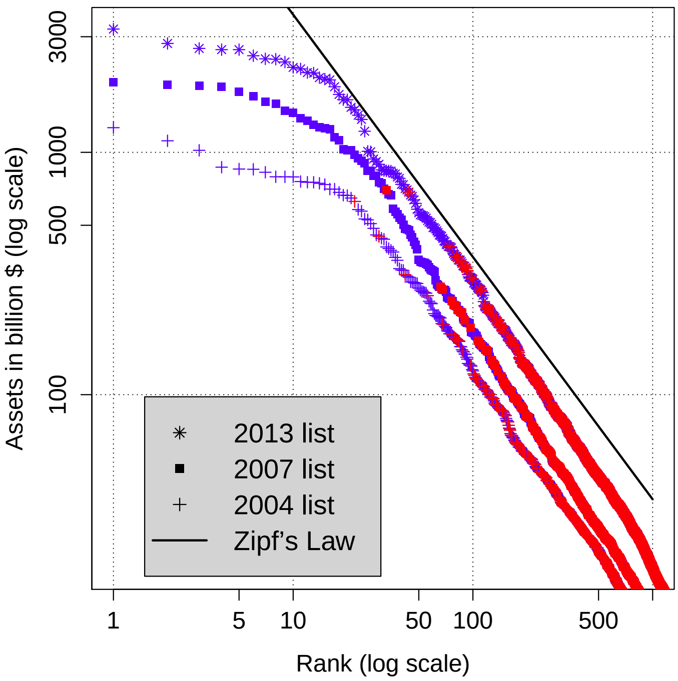

FIGURE 1. Rank plot of the 2004 list (+), 2007 list (□) and 2013 list (∗) of FG2000 by asset size. Financial firms are shown in blue, while the other firms in red. The straight line corresponds to Zipf's law and is drawn for comparison.

can broadly be described as credit intermediation involving entities and activities outside the regular banking system (see Financial Stability Board (2012), p. 3). Section 4 concludes the paper.

2. Proportional Random Growth Model

The observed Paretian distribution has generally been related to a mechanism of PRG which dictates that firms grow proportionally to their size (see Gabaix (2009), p. 259, for more details). In particular, a key empirical testable hypothesis of PRG models is that the rate of return on assets (i.e. the ratio of total profits on total assets) should be independent of the level of assets, as it should be for industries with constant returns to scale. Firms in the financial sector are expected to obey constant returns to scale. Our analysis of the FG2000 sample corroborates this hypothesis: The left panel in Figure 2 provides evidence on the flat relationship between the rates of return on assets for the years 2011 and 2012 (corresponding to 2012 and 2013 lists of FG2000) and the level of total assets for most of the range of the assets distribution[6]. In spite of this behavior of the rates of return on assets, the expected level of (relative) assets of firms at period t conditioned to the level of (relative) assets at period $t-1$, is proportional to the latter only in an intermediate range. As shown in the right panel of Figure 2, while banks of intermediate size grow proportionally to their size, the largest ones grow less than linearly (see caption of the figure). These findings are consistent with earlier results in Wheelock and Wilson (2012) and Restrepo et al. (2013).

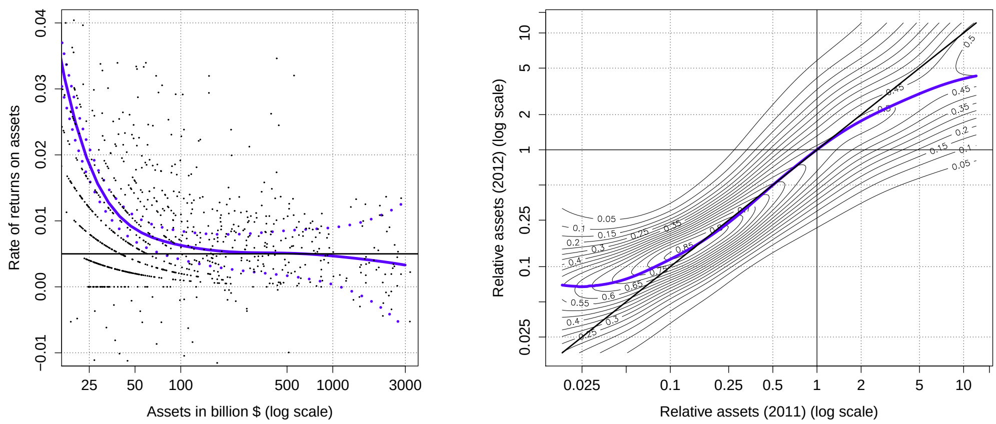

FIGURE 2. Left panel: nonparametric estimate of the relationship between the rates of return on assets and the levels of assets (bold line, dotted lines refer to 5% confidence interval). Right panel: estimate of the stochastic kernel (i.e. of the conditional probability distribution) and of the expected level of assets in 2012 conditioned to the level of assets in 2011 (light grey lines and bold blue line respectively). Both plots refer to financial firms in the 2012 and 2013 FG2000 lists.

While the estimate of the rate of returns shows no evidence of decreasing returns to scale for financial firms, lending support to PGR mechanism, the bending in the estimated expected level of assets highlights how PRG inexplicably does not hold for the largest financial firms (more or less the top 13% in the 2013 list). This finding is reflected in the distribution of asset sizes of financial firms, reported in the left panel of Fig. 3, that follows a power law distribution in an intermediate range, but consistently bend downwards in the top tail. Such deviation from a theoretical power law behavior is much sharper than that occurring in the distribution of all firms (reported in the right panel of Fig. 3).

[6]The nonparametric estimate reported in the left panel of Figure 2 (a Nadaraya-Watson kernel regression) is made by package sm (2013). The estimate of the stochastic kernel (i.e. of the

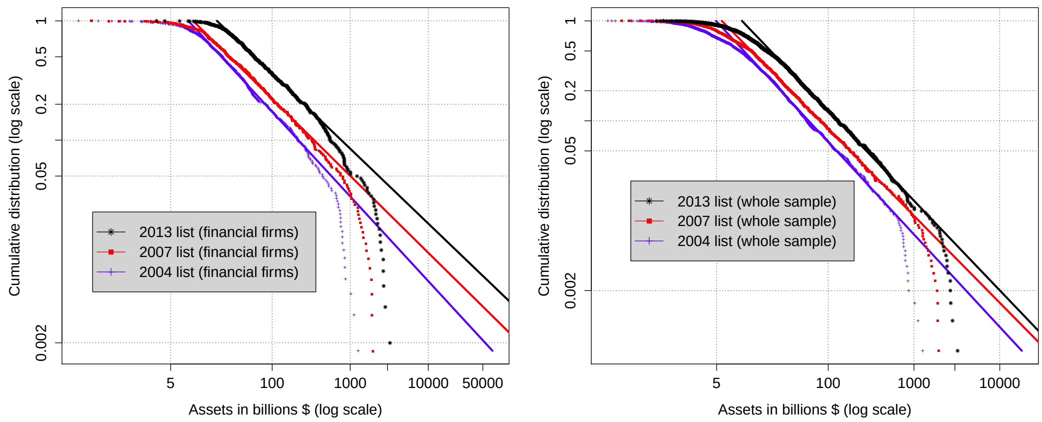

FIGURE 3. Cumulative distribution Prob$\{S \geq x\}$ of asset sizes S for financial (left panel) and all (right panel) firms in 2003, 2006, and 2012 (2004, 2006, and 2013 of FG2000 lists). The straight line is obtained as a linear fit in an intermediate range of $\log \text{Prob}\{S \geq x\}$ vs $\log x$ (see Table 1).

Table 1 reports the ranges considered in the estimate of the power law distribution, and the estimate of the Pareto exponent γ of Eq. (1) for all firms in the FG2000 list from 2004 to 2013 (2005 is missing for lack of data). The estimated Pareto exponent $\hat{\gamma}$ for the whole sample is highest at the beginning of the period and steadily decreases until it reaches the lowest level in 2007 (2008 list), before the great financial crisis. Then it increased suddenly in 2008 and remained relatively stable thereafter. It is worth remarking that the estimated exponent $\hat{\gamma}$ is less than one in the whole period. The simplest PRG model predicts a Pareto exponent larger than one; however Bouchaud and Mezard (2000) argue that $\gamma < 1$ can be obtained within models of PRG with random shocks and redistribution if the flows of assets among firms are restricted in size and happen within a sparse network. Table 1 also reports the estimate $\hat{\gamma}_{\text{fin}}$ of the Pareto exponent of the distribution of financial firms: $\hat{\gamma}_{\text{fin}}$ exhibits a similar behavior to $\hat{\gamma}$, with the important exception that it started again to decline after the crisis.

These findings indicate that the Pareto distribution of asset sizes should be considered as a property that applies to the whole economy, rather than to a particular sector. This is consistent with empirical findings e.g. in Axtell (2001), and suggest that in the absence of anomalies, one would expect an hypothetical assets distribution that would perfectly obey the PRG predictions up to the largest firms.

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conditional probability distribution) in the right panel of Figure 2 is made using the *adaptive kernel estimation* discussed in Fiaschi and Romanelli (2009).

List FG2000	S_-	S_+	$\hat{\gamma}$	$\hat{\gamma}_{\text{fin}}$
2004	14.88	665.14	0.926	0.710
			(0.0012)	(0.0019)
2006	11.02	897.85	0.889	0.678
			(0.0005)	(0.0013)
2007	12.18	992.27	0.871	0.645
			(0.0005)	(0.0012)
2008	12.18	1096.63	0.864	0.655
			(0.0006)	(0.0016)
2009	14.88	1339.43	0.899	0.672
			(0.0008)	(0.0012)
2010	14.88	1339.43	0.891	0.674
			(0.0008)	(0.0011)
2011	18.17	1339.43	0.899	0.669
			(0.0006)	(0.0013)
2012	24.53	1635.98	0.905	0.648
			(0.0009)	(0.0012)
2013	24.53	1998.20	0.897	0.627
			(0.0008)	(0.0009)

Table 1. The range $[S_-, S_+]$ where the power law behavior is estimated (for the whole sample), and the estimated Pareto exponents $\hat{\gamma}$ both for the whole sample and limited to the financial firms in the FG2000 lists from 2004 to 2013 (data for 2005 are not available). Standard errors of the estimated Pareto exponents are reported in brackets.

3. The Shadow Banking Index

Shadow banking (SB) is a relatively new concept; the term itself is attributed to McCulley (2007). SB is a part of the wholesale money market where, in contrast to regular banking, it is not the central bank, but, at least in theory, private institutions that provide a backstop when necessary. This explains why SB has remained outside regulation (see however Fein (2013)). During the 2007-08 crisis, which is often described as a run on the SB system, this private guarantee proved insufficient, and without a massive public intervention the collapse of the SB system would have brought down the whole global finance. The first taxonomy of the different institutions and activities of SB was given by Pozsar (2008), who also constructed a map to describe the flow of assets and funding within the system. The rise of a large part of SB was motivated by regulatory and tax arbitrage, and as such represented the answer of the finance industry to regulation, in particular to capital requirements. Other components responded to real economic demand, and have an important function, see Mehrling et al. (2013). Irrespective of the shortcomings or merits of the system, it is still true that shadow banking has remained by and large unregulated, its systemic risks implications uncharted and its connections opaque.

For us, the only property of interest of the SB system is its total volume. Estimates of its size differ markedly, in particular for the difficulty to precisely individuate which financial activities should be included in the calculation. For example, the Deloitte Shadow Banking Index (see Deloitte (2012)) shows a rise of the SB system in the US before 2008, but then it displays a dramatic drop, suggesting that now the phenomenon is over. The index is built from specific components which are known to have played a major role in the crisis and its decline after 2008 reflects the deflation of these markets. The Financial Stability Board (FSB) estimates that SB "[...] grew rapidly before the crisis, rising from \$26 trillion in 2002 to \$62 trillion in 2007. The size of the total system declined slightly in 2008 but increased subsequently to reach \$67 trillion in 2011" (see Financial Stability Board (2012)). However, as observed by Adrian et al. (2013) "[...] the shadow banking system comprises many different entities and activities. In addition, the types of entities and activities which are of particular concern will change in the future, in response to new regulations".

Below we propose an index for the size of the SB system, denoted by I_{SB}, based on the idea that, in an ideal economy where finance operates in a regime of constant returns to scale, the power law distribution should extend all the way to the largest firms. Since the top tail of the distribution is dominated by financial firms, we are led to attribute the mass missing from the distribution of asset sizes to "[...] credit intermediation involving entities and activities outside the regular banking system" (see Financial Stability Board (2012)), i.e. to shadow banking. Fitting the middle range of the distribution to a power law behavior (as in the left panel of Fig. 3) leads us to a theoretical estimate $\hat{S}_{[k]}$ of what the size of the k^{th} largest firm should be. Summing the difference between the latter and the actual size $S_{[k]}$ of the k^{th} largest firm, over k, i.e.:

$$I_{SB} = \sum_{k=1}^{N} \left(\hat{S}_{[k]} - S_{[k]} \right) \tag{2}$$

provides our estimate of the size of the SB system.

A comparison between Table 1 and Fig. 4 shows how I_{SB} is strongly (anti)correlated with $\hat{\gamma}_{\text{fin}}$ (but not with $\hat{\gamma}$): when the assets distribution of financial firms gets broader (i.e. γ_{fin} decreases), I_{SB} increases and *vice-versa*. As expected, the behavior of financial firms is therefore at the core of the dynamics of I_{SB}.

Fig. 4 reports for comparison also the estimated size of the SB system by FSB (see Financial Stability Board (2012)). Both the latter and I_{SB} show a strong rise before the crisis in 2007, a drop in 2008 (much more severe for I_{SB}), and a growth after 2008, but with I_{SB} increasing at a faster pace, especially in 2011. We guess that the estimate by FSB for the more recent years could not include components

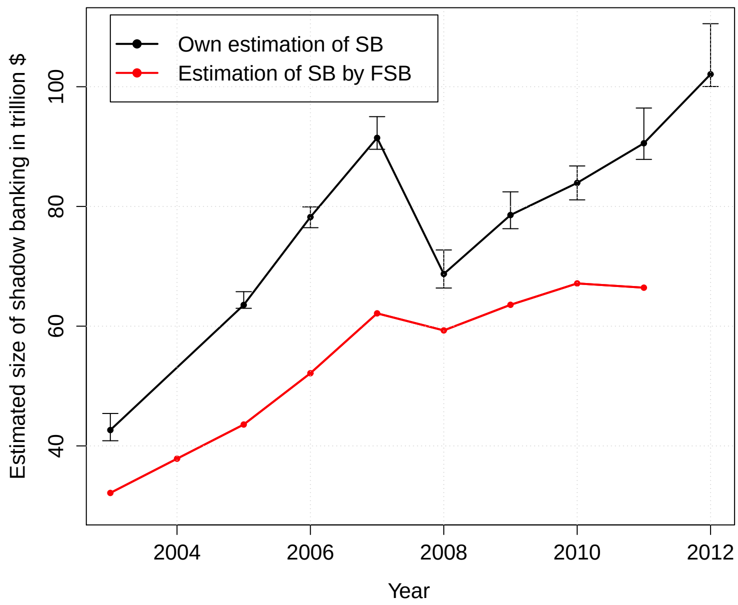

FIGURE 4. Comparison between our index of SB I_{SB} with the estimate of the size of SB made by FSB (Financial Stability Board (2012)) for the period 2003-2012. The reported confidence bands for our estimate of SB are calculated on the base of ± 2 standard errors in the estimate of coefficients of power law distributions.

of SB activities not yet known or well understood (see the next section for more details).

A few comments are in order about I_{SB}:

- I_{SB} is a genuine systemic indicator, as it depends on a collective property of the economy. It is hard to manipulate and simple to compute, as it requires only data publicly available.
- I_{SB} does not rely on a detailed list of activities which contribute to the SB system; it is therefore robust to change in regulation and fiscal policy.
- I_{SB} implicitly attributes SB activities to the largest financial firms which populate the top tail of assets distribution. It is well documented that the main financial firms originated most of the SB activities before the crisis (see Fein (2013)). Yet, I_{SB} also crucially depends on the exponent γ, whose estimate depends on the shape of the distribution in the intermediate range. In particular, *ceteris paribus*, I_{SB} is expected to increase if the exponent γ decreases and *vice-versa*.

4. Conclusions and Outlook

This paper takes a non-standard approach to study the systemic properties of an economy. On one side, based on solid evidence in the literature (Gabaix (2009)), we consider the Pareto distribution for asset sizes as an empirical law of an economy. This empirical law arises from a generic mechanism – proportional random growth – that is expected to work in particular for financial firms. The actual distribution of firm sizes, at the global scale, follows closely this empirical law in the middle range, but it deviates markedly from it in the upper tail, which is populated entirely by financial firms. We invoke SB as the element that would reconcile observations with the expected law. This allows us to derive an index that identifies the size of SB with the missing mass in the top tail of the asset size distribution. This index is based on simple and robust statistical features, that are expected to characterize the collective behavior of an economy. Haldane and Madouros (2012) recently suggested that the increase in complexity of financial markets should be tamed by measures based on simple metrics rather than on a detailed description and classification of financial activities, which are unlikely to keep pace with the rate of innovations in the financial industry. The index of SB proposed in this paper is indeed a contribution in this direction.

Our study also raises a number of issues. We conjecture that the disparity between our index of SB and the estimate reported by FSB may be attributable to China's financial sector. Chinese firms are rapidly growing in an environment that, in turn, is also changing very quickly, with features not always transparent or well understood (see, e.g., Yao (2013)). On the theoretical side, it would be interesting to investigate minimal modifications of PRG models, such as that proposed by Bouchaud and Mezard (2000) that could reproduce the observed behavior of the largest financial firms. The faster growth of financial firms with respect to non-financial firms may be related, following Marsili (2013), to the proliferation of financial instruments. It would be intersting to relate these theoretical approaches in order to reconcile empirical evidence with theoretical models and shed light on the rôle that finance is playing in our global economy.

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